Presentation Of Goals

(1) Discuss the international trends in math, and reasons why the United States lags behind some industrialized nations in math and science.
(2) Explore the role of various cognitive constructs including working memory, visual-spatial functioning, language, and executive functioning, with respect to math problem solving ability.
(3) Discuss the main neural pathways that contribute to the development of number sense and quantitative reasoning.
(4) Discuss three subtypes of math disabilities, and specific remediation strategies for each type.
(5) Introduce the Fam, a diagnostic test of mathematics designed to examine the underlying neurodevelopmental processes that support the acquisition of proficient math skills.

Future Reading Materials

www.schoolneuropsychpress.com
or
@schoolneuropsychpress
4 Common Fallacies Associated with Math

(1) Math abilities are a by-product of IQ and formal education.
- Approximate number system - emerges extremely early in development and is a mental representational system used to approximate a given amount. Distinguishes math LD when IQ controlled for (Mazzocco, Feigenson, & Halberda, 2011).
- "Calendrical" Calculations???

(2) Math is a right hemispheric task.
- Multiple neural networks are involved in the processing of stored quantitative knowledge utilizing both hemispheres (Dehaene & Cohen, 1997; Geary, 2004; Ashkenazi et al., 2013; Skagerlund & Traff, 2014).

(3) Boys outperform girls in math.
- "There just aren't gender differences anymore in math performance," says University of Wisconsin-Madison psychology professor Janet Hyde. Based upon a UW-Madison and University of California, Berkeley (2008), researchers report data from more than 7 million students in 10 states from No Child Left Behind Scores. They then calculated the "effect size" of being between .01 and .06....which is not significant.
- Males over-represented at both high and low end of the distribution (Casey, et al, 1997).

(4) Math is independent of language.
- Math is interdependent on language!!

2015 NAEP DATA: 4th & 8th
http://www.nationsreportcard.gov/reading_math_2015/#?grade=4

- 279,000 public and private school 4th grade students in 2015, and 273,00 8th graders from all 50 states (every 2 years).

Content items: overlap with common core (adopted by 43 states)
- Number properties and operations measures students’ understanding of ways to represent, calculate, and estimate.
- Measurement assesses students’ knowledge of measurement for such attributes as capacity, length, area, volume, time, and angles.
- Geometry measures students’ knowledge and understanding of shapes in two and three dimensions and relationships between shapes such as symmetry and transformations.
- Data analysis, statistics, and probability measures students’ understanding of data representation, characteristics of data sets, experiments and samples and probability.
- Algebra measures students’ understanding of patterns, using variables, algebraic representation, and functions.
2015 NAEP DATA: 4th grade

40% of 4th graders at or above a Proficient level in mathematics.

State by State Comparison of Scores

Highest States
- Massachusetts 251
- Minnesota 250
- New Hampshire 249
- Indiana 248
- Wyoming 247
- Virginia 247
- New Jersey 245
- Washington 245
- North Dakota 245

Lowest States
- Alabama 231
- New Mexico 231
- District of Columbia 231
- California 232
- Nevada 234
- Mississippi 234
- Louisiana 234
- West Virginia 235
- Arkansas 235

* National 4th grade average: 240

2015 NAEP DATA: 8th grade

33% of 8th graders at or above a Proficient level in mathematics.
### 2015 NAEP DATA: 8th grade

#### State by State Comparison of Scores

<table>
<thead>
<tr>
<th>Highest States</th>
<th>Lowest States</th>
</tr>
</thead>
<tbody>
<tr>
<td>Massachusetts 297</td>
<td>District of Columbia 263</td>
</tr>
<tr>
<td>New Hampshire 294</td>
<td>Alabama 267</td>
</tr>
<tr>
<td>Minnesota 294</td>
<td>Louisiana 268</td>
</tr>
<tr>
<td>New Jersey 293</td>
<td>Mississippi 271</td>
</tr>
<tr>
<td>Vermont 290</td>
<td>New Mexico 271</td>
</tr>
<tr>
<td>Wisconsin 289</td>
<td>West Virginia 271</td>
</tr>
<tr>
<td>North Dakota 288</td>
<td>Oklahoma 275</td>
</tr>
<tr>
<td>Virginia 288</td>
<td>Arkansas 275</td>
</tr>
<tr>
<td>Montana 287</td>
<td>Florida 275</td>
</tr>
<tr>
<td>Indiana 287</td>
<td>Nevada 275</td>
</tr>
</tbody>
</table>

* National 8th grade average: 281

---

### Is Common Core at Fault?

#### Key Learning Shifts:

1. Greater focus on fewer topics:
   - **K-2:** Addition and Subtraction concepts
   - **3-5:** Multiplication, Division, Fraction concepts
   - **6th:** Ratios, Proportions, Algebraic expressions
   - **7th:** Ratios, Proportions, and Advanced Algebraic expressions
   - **8th:** Linear Algebra and Linear Functions

2. Greater coherence to connect mathematics around coherent progression from grade to grade. Too often, math is taught as discrete skills independent of one another, and made up of short-cuts that rob students of core number sense.

3. Increase rigor through a combination of teaching greater conceptual knowledge, in addition to procedural fluency skills (automaticity), and the ability to apply mathematics to problem solving situations.

* Problems with implementation, developing texts, and teacher training.

---

### PISA DATA (2012): 15 yr. olds

*Program for International Student Assessment*

* A test of mathematical literacy for 15-year old students which focuses upon the direct application of mathematical principles. The test is administered every three years, with 65 countries participating in 2012. The test was not designed to measure curricular outcomes, but rather to assess mathematics literacy within a real world context.

#### Content Samples:

- **Change and relationship:** Can students model change and relationships with the appropriate functions and equations?
- **Space and shape:** Can students understand perspective, create and read maps, and manipulate 3D objects?
- **Quantity:** Are 15-year-olds able to comprehend multiple representations of numbers, engage in mental calculation, employ estimation, and assess the reasonableness of results?
- **Uncertainty and data:** Can students use probability and statistics and other techniques of data representation and description to mathematically describe, model, and interpret uncertainty?
In 2012, the average U.S. score in mathematics literacy was 481, lower than the international average score of 494.

Among the 65 countries in the sample, the U.S. was outperformed by 26 countries, and 9 countries had average scores not measurably different.

There was no measureable change in the U.S. position when compared to the international average in both 2003 to 2006. The overall score dropped from 2009 (487).

Massachusetts (514) and Connecticut (506) highest scores, while Florida (467) was lowest.

### PISA DATA (2012): 15 yr. olds

<table>
<thead>
<tr>
<th>Country</th>
<th>Average Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>International Average</td>
<td>494</td>
</tr>
<tr>
<td>1. Shanghai - China</td>
<td>613</td>
</tr>
<tr>
<td>2. Singapore</td>
<td>573</td>
</tr>
<tr>
<td>3. Hong Kong - China</td>
<td>561</td>
</tr>
<tr>
<td>4. Chinese Taipei</td>
<td>560</td>
</tr>
<tr>
<td>5. Korea</td>
<td>554</td>
</tr>
<tr>
<td>6. Macao - China</td>
<td>538</td>
</tr>
<tr>
<td>7. Japan</td>
<td>536</td>
</tr>
<tr>
<td>8. Liechtenstein</td>
<td>535</td>
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<tr>
<td>9. Switzerland</td>
<td>531</td>
</tr>
<tr>
<td>10. Netherlands</td>
<td>523</td>
</tr>
<tr>
<td>11. Estonia</td>
<td>521</td>
</tr>
<tr>
<td>12. Finland</td>
<td>519</td>
</tr>
<tr>
<td>13. Canada</td>
<td>518</td>
</tr>
<tr>
<td>14. Poland</td>
<td>518</td>
</tr>
<tr>
<td>15. Belgium</td>
<td>515</td>
</tr>
<tr>
<td>16. Germany</td>
<td>514</td>
</tr>
<tr>
<td>17. Vietnam</td>
<td>511</td>
</tr>
<tr>
<td>18. Austria</td>
<td>506</td>
</tr>
<tr>
<td>19. Australia</td>
<td>504</td>
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<tr>
<td>20. Ireland</td>
<td>501</td>
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<tr>
<td>21. Slovenia</td>
<td>501</td>
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<tr>
<td>22. Denmark</td>
<td>500</td>
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<tr>
<td>23. New Zealand</td>
<td>500</td>
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<tr>
<td>24. Czech Republic</td>
<td>499</td>
</tr>
<tr>
<td>25. France</td>
<td>495</td>
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<tr>
<td>26. United Kingdom</td>
<td>494</td>
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<tr>
<td>27. Iceland</td>
<td>493</td>
</tr>
<tr>
<td>28. Latvia</td>
<td>491</td>
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<tr>
<td>29. Luxembourg</td>
<td>490</td>
</tr>
<tr>
<td>30. Norway</td>
<td>489</td>
</tr>
<tr>
<td>31. Portugal</td>
<td>487</td>
</tr>
<tr>
<td>32. Italy</td>
<td>485</td>
</tr>
<tr>
<td>33. Spain</td>
<td>484</td>
</tr>
<tr>
<td>34. Russian Federation</td>
<td>482</td>
</tr>
<tr>
<td>35. Slovak Republic</td>
<td>482</td>
</tr>
<tr>
<td>36. United States</td>
<td>481</td>
</tr>
</tbody>
</table>
Given the global demand for high tech workers, there is a greater exportation of jobs overseas due to a combination of cheaper wages, as well as a better educated workforce in mathematics and science.

4 Reasons for U.S Decline

1. **The language of math matters**! Building number connections centered around a base-10 principle is crucial in the development of mathematical efficiency when problem solving.

2. **Dry and boring material**. Mathematical skill building needs to be **FUN**, and therefore needs to be presented in the format of games and activities.

3. **Too much focus on the answers**. In order to become facilitators of mathematical knowledge, students should practice multiple methods of problem solving from both a visual-spatial and verbal approach.

4. **Time on task**. Most elementary math instruction occurs in the afternoon, just 45 minutes per day.

What is a Math Disability?

* **Dyscalculia** - children with specific math-related deficits, including:
  a) Learning and retrieving mathematical facts
  b) Executing math calculation procedures
  c) Basic number sense and concept development

* Involves both cognitive and quantitative processes that underscore the development of mathematical achievement. (Rosselli, Matute, Pinto, & Ardila 2006).

**Math Learning Disability (MLD)** - a generic term referring to children whose math performance in the classroom is substantially below age- and grade-level expectations. Often used when there is unexpected underachievement.

* Approximately 6-14% of school age children have persistent difficulty with math (Manserco, Feigenson & Halberda, 2011)
The “MLD” Profile
(Geary, 2011; Rasanen, et al., 2009)

1. Are slower in basic numeric processing tasks:
   - Rapidly identifying numbers.
   - Making comparisons between magnitude of numbers.
   - Counting forwards and backwards

2. Struggle in determining quantitative meaning of numbers:
   - Poor use of strategies.
   - Do not visualize numbers well.

3. Have difficulty learning basic calculation procedures needed to problem solve.

MLD Error Profile:
- Prone to procedural errors such as saying “5,6,7” when solving $5 + 3 = ___$
- Misalign numbers: $\begin{array}{c} 36 \\ + 3 \\ \hline 66 \end{array}$
- Fail to borrow in a sequential manner: $\begin{array}{c} 83 \\ - 44 \\ \hline 41 \end{array}$
- Often deploy the wrong computational process: “The school store sold twice as many pencils to Sam than Robert. If Sam was sold four pencils, how many pencils were sold to Robert?” $\boxed{？}$
- Poor retrieval of basic facts: $7 \times 6 = 35$

The Neural Machinery of Mathematics

Language Skills: (temporal lobes)
- Most Asian languages have linguistic counting systems past ten (ten-one, ten-two, etc) whereas English deviates from base-10 system (Campbell & Xue, 2001).
- In English counting system, decades come first then unit (i.e. twenty-one) or sometimes this is reversed (i.e. fifteen, sixteen, etc...)
- Chinese numbers are brief (i.e. 4=si, 7=qi) allowing for more efficient memory. By age four, Chinese students can count to 40, U.S. students to 15.
- U.S. kids spend 180 days in school
  - South Korea children spend 220 days in school
  - Japan kids spend 243 days in school
The Neural Machinery of Mathematics

**Language Skills:** (temporal lobes)
- Early math skills tend to be verbally encoded, and that is how we initially learn math facts.
- Children with math disabilities frequently have delays in their language development (Shalev et al., 2000)
- Word problems offer an intricate relationship between language and mathematics. Terms such as *all, some, neither, sum, etc.* may be confusing when embedded in the grammatical complexity of word problems (Levine & Reed, 1999).

Working Memory In The Brain

<table>
<thead>
<tr>
<th>Working Memory System</th>
<th>Mathematical Skill</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Phonological Loop</strong></td>
<td>Retrieval of math facts</td>
</tr>
<tr>
<td><strong>Visual-Spatial Sketchpad</strong></td>
<td>Writing dictated numbers</td>
</tr>
<tr>
<td><strong>Central Executive System</strong></td>
<td>Mental math</td>
</tr>
<tr>
<td></td>
<td>Magnitude comparisons</td>
</tr>
<tr>
<td></td>
<td>Geometric Proofs</td>
</tr>
<tr>
<td></td>
<td>Inhibiting distracting thoughts</td>
</tr>
<tr>
<td></td>
<td>Modulating anxiety</td>
</tr>
<tr>
<td></td>
<td>Regulating emotional distress.</td>
</tr>
</tbody>
</table>

Horizontal Vs. Vertical

**Trbovich & LeFevre, 2003.**
- Solving problems in a vertical format required the use of more visual resources, particularly the visual-spatial sketchpad of working memory.
- Solving problems in a horizontal format required more phonological resources resulting in slower performance.
Interventions for Lower Working Memory

- Number-line situated on student's desk.
- Use a calculator.
- Reduce anxiety in the classroom.
- Increase number sense through games such as dice, domino's, cards, etc.
- Encourage paper and pencil use while calculating equations.
- Use mnemonic techniques to teach math algorithm's and sequential steps to problem solving (i.e. The steps for long division are Divide, Multiply, Subtract, Bring Down: Dad Mom Sister Brother Read, Monkeys, Smell Dad)

The Neural Machinery of Mathematics

Executive Functioning Skills: (frontal lobes)

- Executive control mechanisms are a set of directive processes such as planning, self-monitoring, organizing, and allocating attention resources to effectively execute a goal directed task.
- Executive functioning dictates "what to do when", a critical process in solving word problems.
- Executive functioning allows students to choose an appropriate algorithm when problem solving.
The Neural Machinery of Mathematics

**EXECUTIVE DYSFUNCTION**

- Selective Attention
  - Anterior Cingulate/Subcortical structures
  - Poor attention to math operational signs
  - Place value mis-aligned

- Planning Skills
  - Dorsal-lateral PFC
  - Selection of math process impaired
  - Difficulty determining salient information in word problems

**BRAIN REGION**

**MATH SKILL**

**EXECUTIVE DYSFUNCTION**

- Organization Skills
  - Dorsal-lateral PFC
  - Inconsistent lining up math equations
  - Frequent erasers
  - Difficulty setting up problems

- Self-Monitoring
  - Dorsal-lateral PFC
  - Limited double-checking of work
  - Unaware of plausibility to a response.

**BRAIN REGION**

**MATH SKILL**

**MATH FLUENCY** (Russell, 1999)

- Efficiency: Student does not get bogged down into too many steps or lose track of plan or strategy.
- Flexibility: Knowledge of more than one approach to problem solving. Allows student to choose appropriate strategy and to double check work.
- Accuracy: A working knowledge of number facts, combinations, and other important number relationships.
- FLUENCY: Efficiency, Flexibility, and Accuracy.
Building a Math Brain: 4 Critical Factors

1. **Approximate Number System** - non-symbolic representation of math represented by space and time.

   - A mental representational system of visual-spatial approximations that may underscore "number sense".
   - Emerges independent of instruction (innate) and in non-humans as well. A preverbal skill.
   - Distinguishes math LD from students from typical peers.
   - Intuitively judging which line at the grocery store is shortest, or whether there is enough milk left in the carton to make breakfast are everyday examples.
   - Activation in inferior parietal sulcus.
   

2. **Connectivity** - linking non-symbolic representations with symbolic representations (numerals) to form our own internal number line.

   - Connectivity Hypothesis: Neuroimaging studies of the brain have shown distinct, though overlapping, neural circuits involved with non-symbolic data, and symbolic processing of information (Kucian et al., 2006; Rykhelevskih et al., 2009; Holloway et al., 2011; Ashkenazi et al., 2014).
   - The intraparietal sulcus tends to be involved in the nonsymbolic, or magnitude representation of numbers primarily in the right hemisphere (Botas et al., 2008).
   - The symbolic processing of digits involves the left angular gyrus and inferior frontal gyrus. (Ansari, 2008; Butterworth & Varma, 2014).

3. **Automaticity** - facts and procedures.

4. **Quantitative knowledge** - mathematical reasoning emerges from the development of number sense as students learn to apply mathematics to real world problems.
Mapping the Math Brain

Symbolic vs. Non-symbolic Brain Regions

Measuring Connectivity: The Distance Effect

Distance Effect: refers to the fact that when students are presented with two numerals and asked which one is larger, they tend to respond fastest when the numerals are quantitatively far apart, rather than close together (Butterworth & Varma, 2014).

<table>
<thead>
<tr>
<th>Fact Response</th>
<th>Slow Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 94</td>
<td>6 8</td>
</tr>
<tr>
<td>3 44</td>
<td>12 11</td>
</tr>
<tr>
<td>47 1</td>
<td>31 29</td>
</tr>
<tr>
<td>87 15</td>
<td>56 58</td>
</tr>
<tr>
<td>17 71</td>
<td>19 17</td>
</tr>
<tr>
<td>8 39</td>
<td>81 78</td>
</tr>
</tbody>
</table>

Measuring Connectivity: The Distance Effect

Distance Effect: Whenever both numbers are relatively large, response times tend to be slower and less accurate as well (Weber’s Law).

- Children with developmental dyscalculia tend to respond more slowly than typical peers when making comparisons between two numbers, even when controlling for IQ and general reading ability (Balogh & Toff, 2014).
- A child’s reaction time, tends to be an excellent predictor of math fluency and math fact retrieval skills (Holloway & Ansari, 2009).

<table>
<thead>
<tr>
<th>Fact Response</th>
<th>Slow Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 94</td>
<td>1,211</td>
</tr>
<tr>
<td>3 44</td>
<td>38,004</td>
</tr>
<tr>
<td>47 1</td>
<td>907</td>
</tr>
<tr>
<td>87 15</td>
<td>10,242</td>
</tr>
<tr>
<td>17 71</td>
<td>261,789</td>
</tr>
<tr>
<td>8 39</td>
<td>8,111</td>
</tr>
</tbody>
</table>
3. Math Facts and Brain Organization

- As number processing becomes more automatic, there is a shift of effortful control toward the back of the brain (Ashkenazi et al., 2014; Cho et al., 2012).
- Children with dyscalculia use more inefficient strategies, especially with subtraction (Rosenberg-Lee et al., 2015).
- Typically developing children often show a rapid shift from using slower, effortful counting strategies toward using more direct forms of automated fact retrieval by 3rd grade (Geary, 2004).
- MD children struggle to use verbally mediated strategies modulated by the temporal lobes (reduced activation) when learning facts, which slows down retrieval (Berteletti, Prado, & Booth, 2014).

Mapping the Math Brain

Math fact retrieval and the brain
(Cho et al., 2012)

4. Quantitative Knowledge: The Key for Higher Level Math Skills

- The development of quantitative knowledge is critical to comprehend more complex mathematics, as well as to establish cognitive flexibility when problem solving. This is often dependent upon a variety of neuropsychological constructs including:
  1. Verbal retrieval skills
  2. Procedural and sequential knowledge
  3. Visual-spatial reasoning
  4. Executive functioning

- For example, the ability to transcode challenging mathematical equations into more palatable forms of operations requires good executive functioning skills. Take the equation $9 \times 16 = \underline{\hspace{2cm}}$. 
3 Subtypes of Math Disabilities

(1) **Verbal Dyscalculia Subtype:**
Main deficit is the automatic retrieval of number facts which have been stored in a linguistic code.

- Over-reliance on manipulatives when problem solving.
- Multiplication and addition often impaired.
- Poor at math fluency tests.
- Math algorithms often preserved.
- Often have learning disabilities in language arts as well.

**Key Constructs:** Language & Verbal Retrieval Skills

(2) **Procedural Dyscalculia Subtype:**
A breakdown in comprehending the syntax rules in sequencing and counting numeric information.

- Difficulty recalling the algorithm or sequence of steps when performing longer math operations.
- Confusion with long division and place value.
- Retrieval of math facts such as single digit addition, subtraction, and multiplication, as well as magnitude comparisons often preserved.
- Only partial development of "number sense"

**Key Constructs:** Working Memory and Anxiety

(3) **Semantic Subtype (Visual-Spatial):**
A deficit with non-symbolic representations of math including estimation skills, aligning numbers in columns, magnitude representations, and pattern recognition skills among objects (right hemisphere).

- In the left hemisphere, impacts visual inferencing of verbal information. This may impact applying visual strategies to verbally mediated problems. For example,

  "A laboratory used 120 fence posts in an experiment comparing two types of paint. Six fewer than twice as many fence posts were painted with paint A as were painted with paint B. How many fence posts were painted with paint A? Paint B?"

**Key Constructs:** Visual-Spatial processing
(3) Semantic Dyscalculia Subtype (Conceptual):
A deficit with the symbolic representations of numbers and amounts, as students struggle to comprehend magnitude representations between numbers.

- Poor “number sense” and spatial attention.
- Difficulty evaluating the plausibility of a response (e.g. $2 \times 4 = 24$)
- Inability to transcode math operations into a more palatable form (e.g. $9 \times 4$ is same as $(4 \times 10) - 4$).
- Poor magnitude comparisons.

Key Constructs: Quantitative Reasoning & Executive Functioning

General Dyscalculia Interventions

- Freedom from anxiety in class setting. Allow extra time for assignments and eliminate fluency drills.
- Mnemonic strategies (i.e. long division - Dead Monkeys Smell Bad)
- Talk aloud all regrouping strategies.
- Use graph paper to line up equations.
- Adopt a curriculum such as “Math Investigations” which allows students to select their own algorithm.
- Attach number-line to desk and provide as many manipulatives as possible when problem solving.
- Teach skip-counting to learn multiplication facts.
- Teach base-10 counting strategies.
- Teach patterns and relationships: (Melissa’s Game, Mama’s Game, Cordelia’s Game, Habib’s Game)
- Teach students to think in “pictures” as well as “words”.
- Have students explain their strategies when problem solving to expand problem solving options.
- Teach estimation skills to allow for effective previewing of response.
- Have students write a math sentence from a verbal sentence.
- Develop a FNWS and BNWS to ten, twenty, and thirty without counting back.
- Construct incorrect answers to equations and have students discriminate correct vs. incorrect responses.
- Reinforce the language of math by re-teaching quantitative words such as more, less, equal, sum, altogether, difference, etc... (April’s Game)
- Incorporate money and measurement strategies to add context and relevance.
New Websites for Math

1. Math Is Fun
   http://www.mathsisfun.com/index.htm
   - Math Is Fun is a website that offers some great math resources for k-12 students and teachers. These resources include worksheets, games, exercises, activities, lesson ideas and many more.

2. Illuminations
   http://illuminations.nctm.org/Default.aspx
   - Illuminations is a project designed by the National Council of Teachers of Mathematics (NCTM). It provides access to quality standards-based resources for teaching and learning mathematics, including interactive tools for students and instructional support for teachers.

3. Math Crunch
   http://mathcrunch.com/
   - Math Crunch is a fast way to get help with math on your phone. Submit a problem, instantly connect with an experienced tutor, reach a solution and most importantly... learn about math.

4. Cool Math
   http://www.coolmath.com/
   - Offers explanations for algebra and calculus...but coolmath4kids is website for children under 12 with games and activities.

Evidenced Based Math Curriculums

**Singapore Math** - Emphasis is on building upon math concepts so re-teaching is not needed, and little time devoted to reviewing previously taught skills before new concept taught.
- Flow of information is from Concrete to Pictorial to Abstract.
- The use of Bar-Models, which represent arithmetic quantities by line segments, facilitate understanding eliminate the need of rote memorization of facts.

**Everyday Mathematics** - core curriculum for students in prek-grade 6 that focuses on real-life problem solving, and quantitative thinking skills.
- A balanced curriculum using a variety of approaches to provide multiple learning opportunities of new concepts. Students are encouraged to experiment with various algorithms, once mastery of the “focus” algorithm is established.
Assessment Algorithm for Math

- Intelligence Tests
- Visual-Spatial Functioning
- Working Memory Capacity
- Executive Functioning
- Attention Skills
- Math Skills and Number Sense
- Math Anxiety Scale
- Developmental and School History

Assessment Instruments for Math

**Math:**
- Wechsler Individual Achievement Test - 3rd Edition
- Woodcock Johnson IV Achievement Test
- Kaufman Test of Educational Achievement (KTEA-III)
- Test of Early Mathematics Ability - 3rd Edition (TEMA-3)
- Comprehensive Mathematical Abilities Test (CMAT)
- Test of Mathematical Abilities - 3rd Edition (TOMA-3)
- WRAT-3
- Academic Achievement Battery (AAB)
- KEYMATH-3
- PAL II Mathematics

**Executive Functions:**
- Wisconsin Card Sort Test
- WJIV (Number Series)
- Woodcock Johnson IV (Number Series)
- D-KEFS (Delis-Kaplan Executive Function Scale)
- D-REF (Delis Rating of Executive Functioning)
- BRIEF II
- NPSY II (Animal Sorting, Design Fluency)
- Test of Executive Control
- CAS (Delis-Kaplan Executive Function System)

**Visual-Spatial:**
- SB5 (Visual-Spatial Processing, Quantitative Reasoning)
- DAS (Matrices, Recall of Designs, Pattern Construction)
- WJIV (Visualization)
- NEPSY II (Arrows, Picture Puzzles, Geom Puzzles)
- Rey-Osterrieth Complex Figure Test
- TONI-3/RIAS (NIX Index)/KABC II (Gestalt Closure)

**Working Memory:**
- WISC V (Verbal & Nonverbal Working Memory)
- WISC V Integrated
- D-KEFS (Trailmaking Test)
- Cognitive Assessment System - 2 (Planned Connections)
- Children's Memory Scale (Dot Locations, Sequences)
- Woodcock Johnson IV (Verbal Attention)
- Wechsler Memory Scale (Visual Reproduction & Paired Associate)
- Wide Range Assessment of Memory and Learning - 4 (Verbal Working Memory & Symbolic Working Memory)
- PAL II: Quantitative and Spatial Working Memory

**Attention:**
- Tea-CP
- NPSY II (Auditory Attention and Response Set)
- CAS-2 (Number Detection, Receptive Attention)
- WJIV (Number Pattern Matching)
- KABC II (Number recall)
- Behavior Scales (ACTers, ADDES, Brown, BASC III, Conners’3)
Assessment Summary for Math

1. Verbal Dyscalculia:
   - Slower fact retrieval skills.
   - Difficulty with word problems.
   - Co-morbid reading/writing difficulties

2. Procedural Dyscalculia:
   - Forget math procedures
   - Better with single-digit facts than longer operations.
   - Working memory limitations

3a. Semantic Dyscalculia: (Visual-Spatial):
   - Difficulty aligning math columns.
   - Poor spatial memory.
   - Poor estimation skills.

3b. Semantic Dyscalculia: (Conceptual)
   - Difficulty transcoding math operations
   - Poor magnitude representations (symbols)
   - Poor conceptual knowledge and quantitative thinking

A neurodevelopmental assessment of mathematics
- Pre-K to College (Ages 4-21)
- Normative sample included 1,061 students
- 19 subtests in complete battery
- Diagnoses 3 subtypes of math disorders
- Includes the FAM-S dyscalculia screening battery
- Total Fam index score and 3 math index scores:
  a) Procedural subtype
  b) Verbal subtype
  c) Semantic subtype
- Qualification Level: S or B

Structure of the FAM

<table>
<thead>
<tr>
<th>Index</th>
<th>Subtest</th>
<th>Grade range</th>
<th>Approximate administration time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Procedural Index (PI)</td>
<td>Forward Number Count (FNC)</td>
<td>PK to college</td>
<td>5 minutes</td>
</tr>
<tr>
<td></td>
<td>Backward Number Count (BNC)</td>
<td>K to college</td>
<td>5 minutes</td>
</tr>
<tr>
<td></td>
<td>Numeric Capacity (NCA)</td>
<td>PK to college</td>
<td>3 minutes</td>
</tr>
<tr>
<td></td>
<td>Sequences (SEQ)</td>
<td>PK to college</td>
<td>5 minutes</td>
</tr>
<tr>
<td></td>
<td>Object Counting (OC)</td>
<td>PK to Grade 2</td>
<td>5 minutes</td>
</tr>
<tr>
<td>Verbal Index (VI)</td>
<td>Rapid Number Naming (RNN)</td>
<td>PK to college</td>
<td>1 minute</td>
</tr>
<tr>
<td></td>
<td>Addition Fluency (AF)</td>
<td>K to college</td>
<td>1 minute</td>
</tr>
<tr>
<td></td>
<td>Subtraction Fluency (SF)</td>
<td>K to college</td>
<td>1 minute</td>
</tr>
<tr>
<td></td>
<td>Multiplication Fluency (MF)</td>
<td>Grade 3 to college</td>
<td>1 minute</td>
</tr>
<tr>
<td></td>
<td>Division Fluency (DF)</td>
<td>Grade 3 to college</td>
<td>1 minute</td>
</tr>
<tr>
<td></td>
<td>Linguistic Math Concepts (LMC)</td>
<td>PK to college</td>
<td>6 minutes</td>
</tr>
<tr>
<td>Semantic Index (SI)</td>
<td>Spatial Memory (SM)</td>
<td>PK to college</td>
<td>5 minutes</td>
</tr>
<tr>
<td></td>
<td>Equation Building (EB)</td>
<td>Grade 3 to college</td>
<td>4 to 6 minutes</td>
</tr>
<tr>
<td></td>
<td>Perceptual Estimation (PE)</td>
<td>PK to college</td>
<td>5 minutes</td>
</tr>
<tr>
<td></td>
<td>Number Comparison (NCO)</td>
<td>PK to college</td>
<td>2 minutes</td>
</tr>
<tr>
<td></td>
<td>Addition Knowledge (AK)</td>
<td>K to college</td>
<td>2 minutes</td>
</tr>
<tr>
<td></td>
<td>Subtraction Knowledge (SK)</td>
<td>K to college</td>
<td>2 minutes</td>
</tr>
<tr>
<td></td>
<td>Multiplication Knowledge (MK)</td>
<td>Grade 3 to college</td>
<td>2 minutes</td>
</tr>
<tr>
<td></td>
<td>Division Knowledge (DK)</td>
<td>Grade 3 to college</td>
<td>2 minutes</td>
</tr>
</tbody>
</table>
Dyscalculia Subtypes

- **Verbal** – an inability to use language-based procedures to assist in arithmetic skills. Difficulties with rapid number identification skills, and retrieving stored mathematical facts.

- **Procedural** – a deficit in the ability to count, order, or sequence numbers or mathematical procedures. Often, there are limitations with symbolic working memory and pattern recognition.

- **Semantic** – a core deficit in both visual-spatial and conceptual components of mathematics. Deficits include poor estimation skills, difficulty aligning numbers in columns, poor magnitude representations, and difficulty selecting a particular mathematical strategy to solve real-world problems.

1. The brain is not modular. Instead, there are continuous interactions within gray matter, rather than white matter connecting distinct and distant structures.

2. Spatially related territories have similar functional properties.

3. Cognitive gradients are varied and organized around a hierarchical axis in the brain.
Verbal Gradient of the FAM

Linguistic Math Concepts
(Tertiary)

Addition, Subtraction, Multiplication, Division, Problem Solving
(Secondary)

Rapid Number Naming
(Primary)

Semantic Gradient of the FAM

Equation Bldg Knowledge
(Tertiary)

Perceptual Estimation, Number Comparisons
(Secondary)

Spatial Memory
(Primary)

Procedural Index

1. Forward Number Count - 30 items
2. Backward Number Count - 30 items
3. Numeric Capacity - 16 items
4. Sequences - 39 items
5. Object Counting - 24 items

Basal & Ceiling Rules: 4 correct and 4 incorrect
Forward Number Count (All Grades)

"We are going to do some counting."

Sample Items
"What number comes after 0?"
Correct Answer: 1
"Starting at 4, count forward by fours"
Correct Answer: 4 8 12 16

Backward Number Count (Grades K+)

"Now, we are going to count backward."

Sample Items
"What number comes before 13?"
Correct Answer: 12
"Starting at 50, count backward by fives"
Correct Answer: 50 45 40 35

Numeric Capacity (All Grades)

"I'm going to say some numbers and I want you to repeat them back to me in exactly the same order."

Sample Item
"7 4 3 6 2 9 1"
Sequences (All Grades)

“For these items, I want you to look at the pattern or sequence and tell me what goes in the blank space. You can answer by pointing or telling me.”

Sample Item (Prek-2nd)

Object Counting (PK through Grade 2)

“I’m going to ask you some counting questions.”

Sample Item

“Which child is 3rd in line?”

Verbal Index

1. Rapid Number Naming – Timed subtest: 30 seconds
2. Addition Fluency - Timed subtest: 30 seconds
3. Subtraction Fluency - Timed subtest: 30 seconds
4. Multiplication Fluency - Timed subtest: 30 seconds
5. Division Fluency - Timed subtest: 30 seconds

Basal & Ceiling Rules: 4 correct and 4 incorrect
Rapid Number Naming (All Grades)

"I want you to name some numbers as quickly as you can."
(30 sec)

| 1 2 3 4 5 2 1 4 3 5 2 | 1 2 3 4 5 2 1 4 3 5 2 |
| 6 3 9 4 1 8 2 6 5 7 9 4 6 |
| 3 7 1 9 2 5 3 8 4 6 1 9 2 |
| 1 5 8 3 6 9 7 2 4 8 5 2 1 |
| 2 7 6 9 3 5 1 4 8 5 6 7 3 |
| 5 4 8 2 7 9 3 1 2 6 4 5 7 |
| 9 3 7 4 5 8 6 2 1 7 9 3 8 |

Addition Fluency (Grades K+)

"I want you to answer some addition problems as quickly as you can without skipping any. You do not need to read the problems aloud, only say your answers."

| 2 + 1 | 2 + 2 | 3 + 2 | 3 + 1 | 1 + 1 | 1 + 3 |
| 1 + 4 | 2 + 8 | 4 + 2 | 2 + 7 | 6 + 4 | 2 + 0 |
| 3 + 4 | 9 + 1 | 6 + 1 | 3 + 3 | 1 + 8 | 4 + 1 |
| 3 + 0 | 2 + 4 | 5 + 2 | 9 + 0 | 8 + 2 | 7 + 4 |
| 5 + 6 | 6 + 6 | 5 + 4 | 0 + 7 | 7 + 5 | 7 + 8 |
| 9 + 7 | 9 + 8 | 2 + 9 | 8 + 6 | 3 + 7 | 9 + 5 |
| 3 + 5 | 5 + 5 | 6 + 3 | 0 + 5 | 4 + 9 | 8 + 1 |
| 8 + 7 | 1 + 6 | 2 + 6 | 1 + 7 | 0 + 2 | 5 + 1 |

Subtraction Fluency (Grades K+)

"Now we are going to do the same thing, but with subtraction problems."

| 3 - 2 | 3 - 1 | 5 - 1 | 5 - 2 | 2 - 1 | 4 - 2 |
| 5 - 4 | 4 - 3 | 4 - 1 | 6 - 2 | 5 - 3 | 7 - 4 |
| 9 - 2 | 6 - 4 | 9 - 6 | 8 - 7 | 6 - 5 | 7 - 2 |
| 8 - 4 | 7 - 6 | 8 - 2 | 6 - 0 | 1 - 1 | 8 - 3 |
| 4 - 4 | 6 - 3 | 9 - 4 | 8 - 0 | 0 - 0 | 7 - 1 |
| 10 - 6 | 2 - 0 | 9 - 1 | 8 - 5 | 6 - 1 | 9 - 3 |
| 9 - 5 | 1 - 0 | 7 - 5 | 6 - 6 | 10 - 7 | 8 - 6 |
| 3 - 0 | 3 - 3 | 8 - 1 | 9 - 8 | 10 - 5 | 7 - 3 |
**Multiplication Fluency (Grades 3+)**

"Now we are going to do the same thing, but with multiplication problems."

<table>
<thead>
<tr>
<th>1 × 1</th>
<th>2 × 2</th>
<th>1 × 8</th>
<th>0 × 9</th>
<th>6 × 1</th>
<th>1 × 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 × 0</td>
<td>1 × 4</td>
<td>4 × 2</td>
<td>2 × 3</td>
<td>3 × 1</td>
<td>4 × 4</td>
</tr>
<tr>
<td>2 × 6</td>
<td>5 × 1</td>
<td>8 × 0</td>
<td>7 × 4</td>
<td>6 × 3</td>
<td>1 × 5</td>
</tr>
<tr>
<td>1 × 0</td>
<td>7 × 1</td>
<td>8 × 4</td>
<td>8 × 6</td>
<td>9 × 4</td>
<td>5 × 7</td>
</tr>
<tr>
<td>7 × 9</td>
<td>7 × 3</td>
<td>4 × 5</td>
<td>0 × 0</td>
<td>6 × 5</td>
<td>8 × 8</td>
</tr>
<tr>
<td>6 × 4</td>
<td>4 × 7</td>
<td>5 × 3</td>
<td>3 × 8</td>
<td>6 × 7</td>
<td>3 × 6</td>
</tr>
<tr>
<td>2 × 5</td>
<td>9 × 3</td>
<td>9 × 1</td>
<td>5 × 2</td>
<td>3 × 2</td>
<td>8 × 7</td>
</tr>
<tr>
<td>3 × 3</td>
<td>5 × 8</td>
<td>3 × 4</td>
<td>6 × 9</td>
<td>3 × 5</td>
<td>8 × 2</td>
</tr>
</tbody>
</table>

---

**Division Fluency (Grades 3+)**

"Now we are going to do the same thing, but with division problems."

<table>
<thead>
<tr>
<th>1 ÷ 1</th>
<th>10 ÷ 2</th>
<th>3 ÷ 1</th>
<th>2 ÷ 1</th>
<th>12 ÷ 2</th>
<th>3 ÷ 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 ÷ 4</td>
<td>2 ÷ 2</td>
<td>14 ÷ 2</td>
<td>12 ÷ 6</td>
<td>16 ÷ 4</td>
<td>9 ÷ 4</td>
</tr>
<tr>
<td>9 ÷ 1</td>
<td>15 ÷ 3</td>
<td>8 ÷ 2</td>
<td>6 ÷ 1</td>
<td>18 ÷ 3</td>
<td>4 ÷ 4</td>
</tr>
<tr>
<td>45 ÷ 9</td>
<td>36 ÷ 6</td>
<td>27 ÷ 3</td>
<td>18 ÷ 2</td>
<td>49 ÷ 7</td>
<td>12 ÷ 4</td>
</tr>
<tr>
<td>36 ÷ 4</td>
<td>14 ÷ 7</td>
<td>18 ÷ 6</td>
<td>72 ÷ 8</td>
<td>63 ÷ 9</td>
<td>32 ÷ 4</td>
</tr>
<tr>
<td>6 ÷ 6</td>
<td>15 ÷ 5</td>
<td>36 ÷ 9</td>
<td>21 ÷ 7</td>
<td>24 ÷ 4</td>
<td>81 ÷ 9</td>
</tr>
<tr>
<td>35 ÷ 5</td>
<td>72 ÷ 9</td>
<td>30 ÷ 5</td>
<td>16 ÷ 2</td>
<td>4 ÷ 2</td>
<td>56 ÷ 8</td>
</tr>
<tr>
<td>54 ÷ 6</td>
<td>42 ÷ 7</td>
<td>30 ÷ 6</td>
<td>7 ÷ 7</td>
<td>8 ÷ 8</td>
<td>24 ÷ 6</td>
</tr>
</tbody>
</table>

---

**Linguistic Math Concepts (All Grades)**

"I am going to ask you questions about specific math terms. Please select the best answers from the choices."

"Which one has the fewest stars?"

A. Sum; B. Difference; C. Factor; or D. Tens place

---

3rd – 8th: In the problem 10 – 3 = 7, what is the 7?

A. Sum; B. Difference; C. Factor; or D. Tens place
Semantic Index

1. Spatial Memory - 32 items
2. Equation Building – 30 items
3. Perceptual Estimation – 26 items
4. Number Comparison – Timed subtest: 60 seconds
5. Addition Knowledge – Timed subtest: 60 seconds
6. Subtraction Knowledge – Timed subtest: 60 seconds
7. Multiplication Knowledge – Timed subtest: 60 seconds
8. Division Knowledge – Timed subtest: 60 seconds

Spatial Memory (All Grades)

“T'm going to briefly show you a picture and then ask you to find it again. The picture may be turned or rotated in a new way.”

Sample Item

Target Picture

Equation Building (Grades 3+)

“T'm going to read some math word problems, and I want you to select the equation you would use to solve each problem.”

Sample Item

Alex did 34 push-ups in gym class today. Henry did 6 more push-ups than Alex did. Which equation shows how many push-ups Henry did?

A. 34 x 6  C. 34 + 6
B. 34 + 6  D. 34 - 6
Perceptual Estimation (All Grades)

Sample Item (1-5)
"I'm going to show you some containers filled with different items. I want you to tell me which container has more items without counting them."
(2 sec to respond)

Perceptual Estimation (All Grades)

Items 6-26
"Now I'm going to show you a picture with the exact number of items written below it. Use this picture and number as clues to help you figure out about how many items are in the picture next to it without counting them."

Number Comparison (All Grades)
60 sec
"For each pair, I want you to draw a line through the larger number."

<table>
<thead>
<tr>
<th>106</th>
<th>99</th>
<th>199</th>
<th>17</th>
<th>103</th>
<th>120</th>
<th>57</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>111</td>
<td>116</td>
<td>125</td>
<td>106</td>
<td>105</td>
<td>601</td>
</tr>
<tr>
<td>898</td>
<td>889</td>
<td>2,100</td>
<td>2,015</td>
<td>6,666</td>
<td>6,677</td>
<td>9,879</td>
</tr>
</tbody>
</table>

| | | | | | | |
Addition Knowledge (Grades K+)
60 sec
“I want you to solve some addition problems. The sums are already there, but one number is missing from each number sentence. Your job is to write the correct responses in the spaces provided.”

10 + __ = 19
20 + __ = 21
30 + __ = 19
12 + __ = 25
1 + 3 + __ = 5
2 + 2 + 9 = 13
4 + 2 + 8 = 14
3 + 2 + 6 = 11
3 + 3 + 3 = 18
7 + 3 + __ = 15
3 + 2 + __ = 11
5 + __ + 8 = 17

Subtraction Knowledge (Grades K+)
60 sec
“Now, we are going to do the same thing, but with subtraction problems.”

5 - __ = 1
2 - __ = 0
4 - __ = 3
__ - 1 = 2
__ - 1 = 1
4 - __ = 4
__ - 3 = 2
3 - __ = 0
__ - 2 = 2
__ - 2 = 1
6 - __ = 3

Multiplication Knowledge (Grades 3+)
60 sec
“Now, we are going to do the same thing, but with multiplication problems.”

2 x __ = 10
__ x 1 = 1
1 x __ = 4
__ x 5 = 10
3 x __ = 12
__ x 2 = 12
__ x 3 = 15
__ x 4 = 16
__ x 2 = 20
6 x __ = 36
__ x 3 = 24
7 x __ = 36
10 x __ = 40
8 x __ = 48
1 x __ = 0
__ x 6 = 36
Division Knowledge (Grades 3+)

60 sec

"Now, we are going to do the same thing, but with division problems."

12 ÷ 3 = __
18 ÷ 3 = __
36 ÷ 3 = __
48 ÷ 3 = __

20 ÷ 4 = __
32 ÷ 4 = __
48 ÷ 4 = __
64 ÷ 4 = __

27 ÷ 9 = __
45 ÷ 9 = __
54 ÷ 9 = __
72 ÷ 9 = __

30 ÷ 5 = __
35 ÷ 5 = __
40 ÷ 5 = __
55 ÷ 5 = __

56 ÷ 7 = __
63 ÷ 7 = __
70 ÷ 7 = __
84 ÷ 7 = __

72 ÷ 8 = __
80 ÷ 8 = __
90 ÷ 8 = __
100 ÷ 8 = __

Interpreting the FAM

1. Determine the FAM Total Index Score

2. Determine the FAM Procedural, Verbal, and Semantic Index Scores and compare these scores to the FAM Total Index score.
   a) Absolute Strengths andWeaknesses
   b) Relative Strengths and Weaknesses
   c) Compare each Index Score to each other.

3. Key Subtest Interpretations

4. Relevant behavioral observations

Correlation between FAM Screening Index and FAM Total Index = .83.

Scoring the FAM
Key Observations

- Skipping lines
- Uneven tempo during fluency tasks
- Sacrificing accuracy for speed
- Sacrificing speed for accuracy
- Finger counting
- Using the "ones" strategy
- Dropping back and counting forward
- Verbal counting
- Working out answers

Case Study: Cheyenne

- Cheyenne is an 8 yr old student in 3rd grade
- She is on grade level in all subjects but math
- Does not know most number facts, and seems to struggle with most math concepts.
- Becoming more frustrated
- Has received targeted math interventions all year.
Case Study: Cheyenne

- **Total FAM Score 77  6%  Moderately Below Avg**
- **Key Analysis (Procedural):** NCA > FNC/BNC suggesting poor ability to identify patterns and relationships in numbers.
- **Key Analysis (Verbal):** LMC > Fluency tasks suggesting better conceptual understanding of math when put in context.
- **Key Analysis (Semantic):** Fluency > Knowledge suggesting rote memorization of facts and lack of deeper understanding of number sense.
- **Conclusion:** MLD (Semantic Dyscalculia)

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- Dice, dominoes, unifix cubes, and manipulatives
- Attach number line to her desk
- Present symbolic math equation and have her tell a story about it.
- Practice skip counting on a hundreds chart to learn patterns and relationships.
- Practice decomposing numbers.
- Use vertical number lines to develop magnitude representations.
- Practice recognizing patterns and sequences of numeric and nonnumeric information.
Based upon a neurocognitive model of brain functioning.
Use in conjunction with an academic achievement test.
Saves time because there is no need for cross battery math assessment, since processing is built into the test.
Explains WHY a student is having math difficulty, not just WHERE the student is achieving.
Directly informs intervention decision making.
Can diagnose, screen, or use for progress monitoring.
Puts the "I" back in IEP's!!!